Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset

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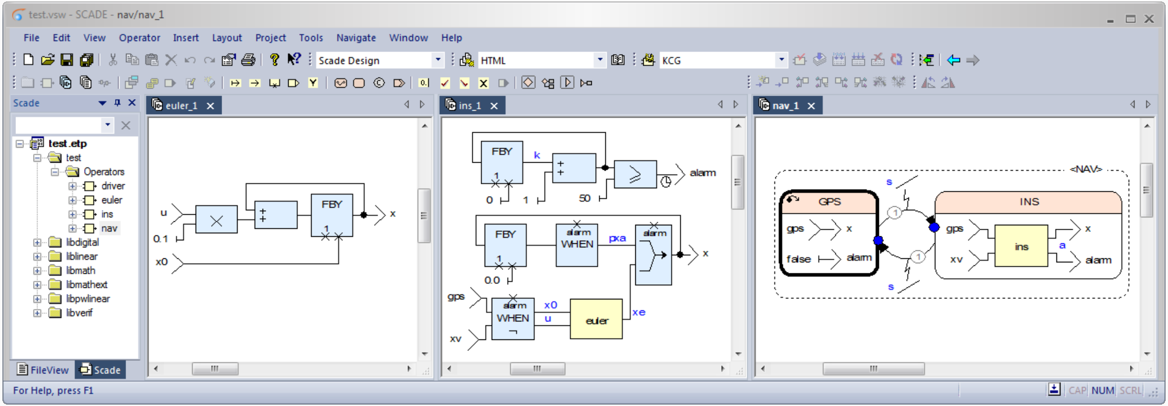
velus.inria.fr
github.com/INRIA/velus

Motivation: Model Based Design in SCADE Suite

www.ansys.com/products/embedded-software/ansys-scade-suite



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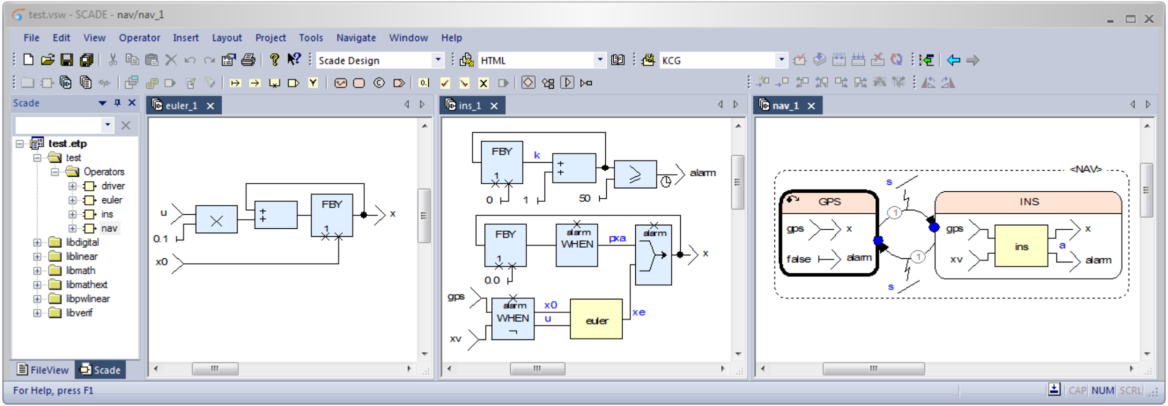

block / node = system

line = signal

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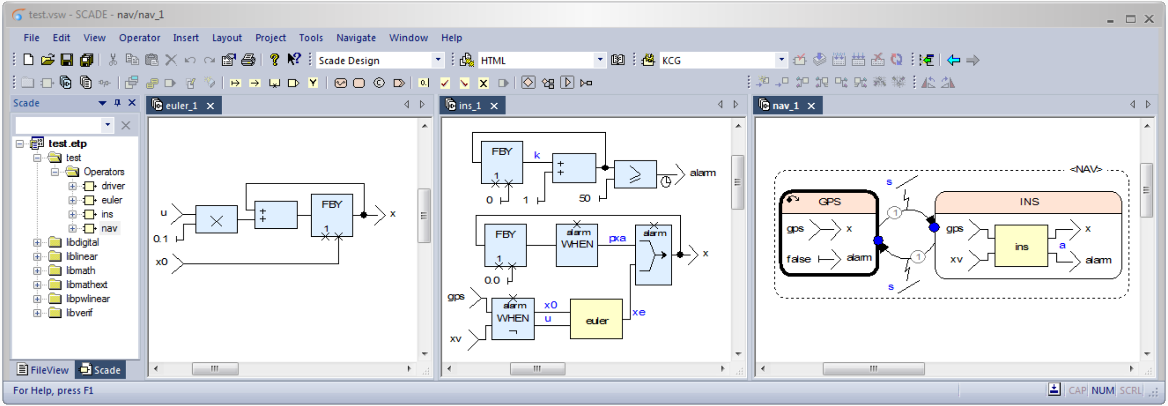

block / node = system = stream function

line = signal = stream of values

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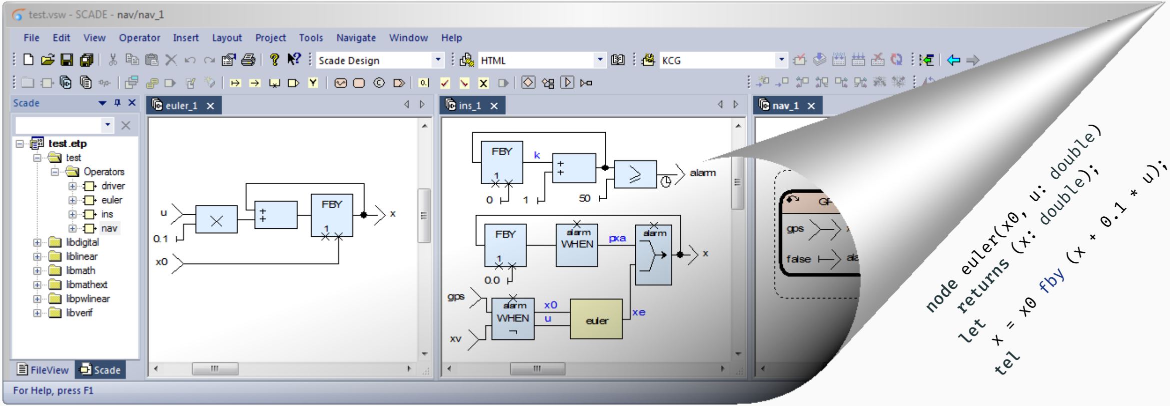

```
node euler(x0, u: double)
  returns (x: double);
let
  x = x0 fby (x + 0.1 * u);
tel
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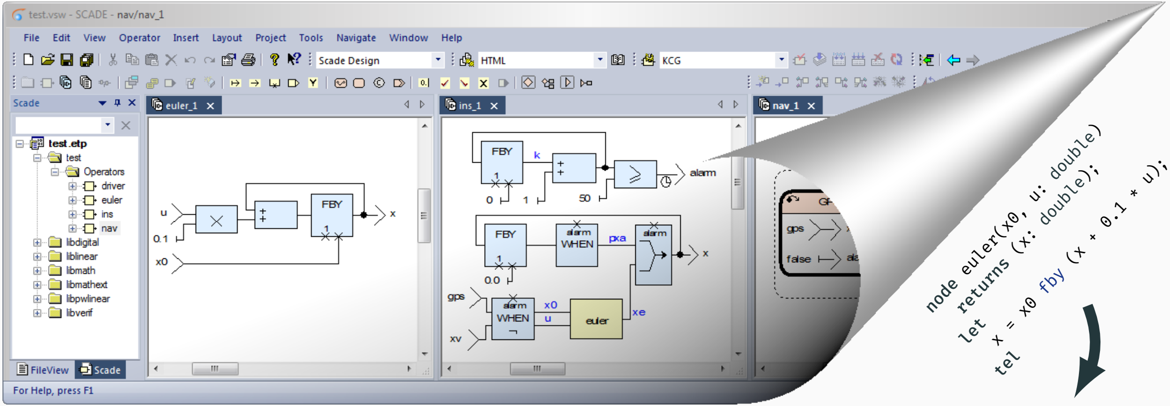

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sequential program
(C, Ada, assembly)

block / node = system = stream function

line = signal = stream of values

The Vélus Project

Model-Based Design Languages

SCADE, Lustre, Simulink

+

Interactive Theorem Provers

Coq

Challenges

1. Mechanize the semantics
2. Prove the compilation algorithms correct

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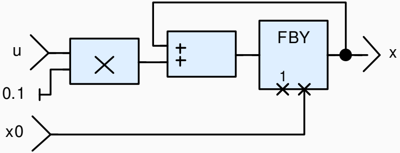
Focus: modular reset

[Caspi et al. (1987); Colaço, Pagano, and Pouzet (2017)]

Example



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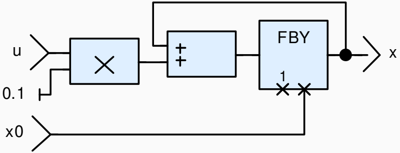
x_0	0.00	1.55	3.62	5.46	$\cdots$
u	15.00	20.00	17.00	12.00	$\cdots$
$x + 0.1 \times u$	1.50	3.50	5.20	6.70	$\cdots$
x	0.00	1.50	3.50	5.20	$\cdots$

Example

[Caspi et al. (1987); Colaço, Pagano, and Pouzet (2017)]



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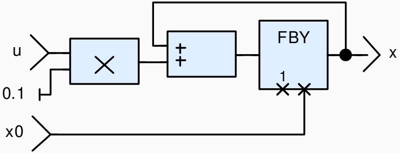

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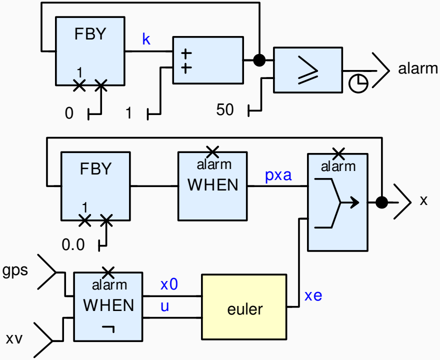

```
node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var pxa, xe: double; k: int;
let
  k = 0 fby (k + 1);
  alarm = (k ≥ 50);
  xe = euler((gps, xv) when not alarm);
  pxa = (0. fby x) when alarm;
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xv	15.00	20.00	17.00	12.00	⋯	18.00	23.00	20.00	⋯
k	0	1	2	3	⋯	49	50	51	⋯
alarm	F	F	F	F	⋯	F	T	T	⋯
xe	0.00	1.50	3.50	5.20	⋯	77.35			⋯
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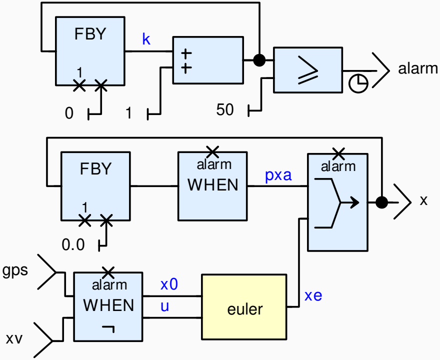

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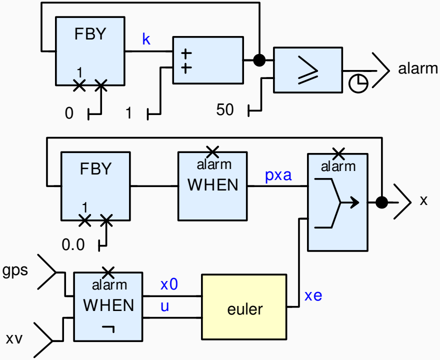

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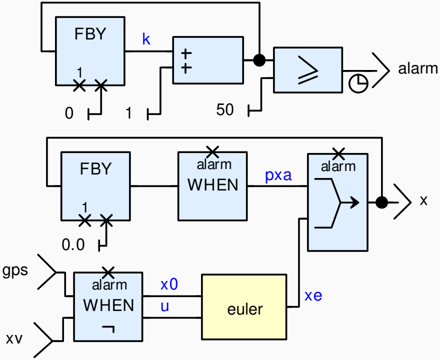

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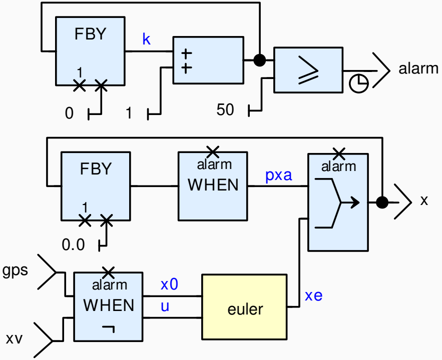

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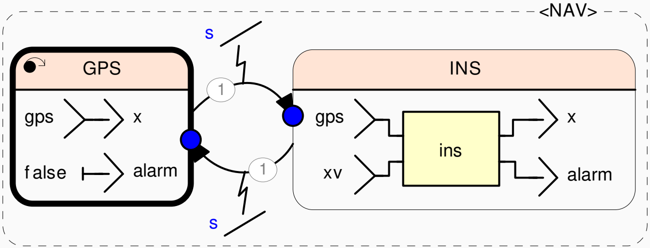
Example

[Colaço, Pagano, and Pouzet (2005)]



Can be compiled into simple constructs

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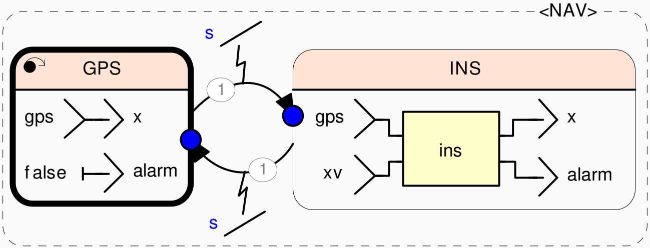

Can be compiled into simple constructs

We need a way to reset the state of a node

Without modular reset

```
node euler(x0, u: double, r: bool)
  returns (x: double);
let
  x = if r then x0 else x0 fby (x + 0.1 * u);
tel

node ins(gps, xv: double, r: bool)
  returns (x: double, alarm: bool)
  var k: int;
let
  x = merge alarm
        ((0. fby x) when alarm)
        (euler((gps, xv, r) whenot alarm));
  alarm = (k ≥ 50);
  k = if r then 0 else 0 fby (k + 1);
tel
...
(x, a) = ins(gps, xv, r);
```

[Caspi (1994); Hamon and Pouzet (2000)]

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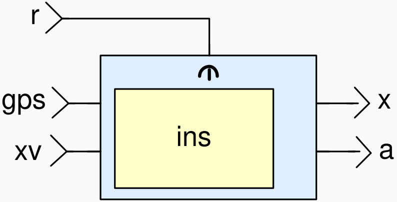
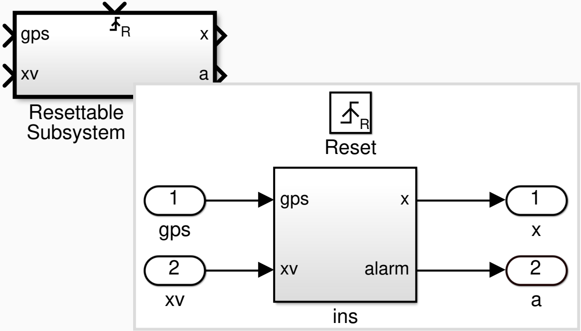
tel
...
(x, a) = (restart ins every r) (gps, xv);
```

Graphical Modular Reset Construct

SCADE

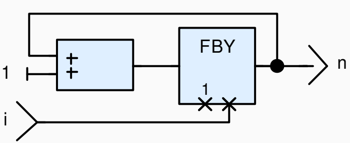


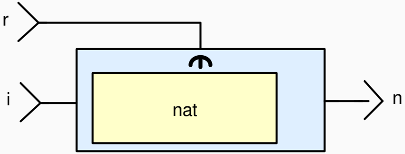
Simulink



A Simpler Example: Intuitive Semantics

```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```





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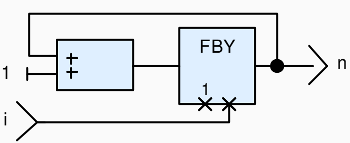

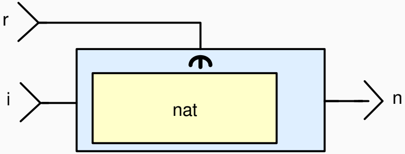

r	F
i	0
$nat(i)$	0
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0

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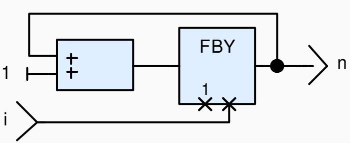

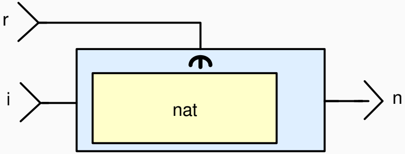

r	F	F
i	0	5
$nat(i)$	0	1
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1

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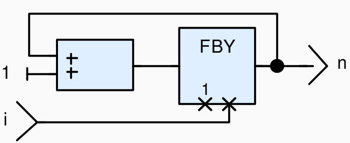

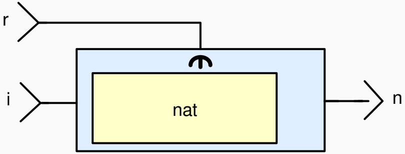

r	F	F	T
i	0	5	10
nat(*i*)	0	1	2
(**restart** *nat* **every** *r*)(*i*)	0	1	10

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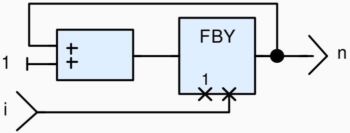

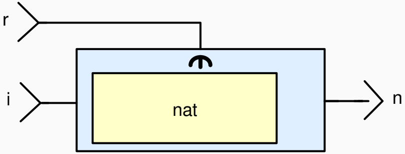

r	F	F	T	F
i	0	5	10	15
$nat(i)$	0	1	2	3
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1	10	11

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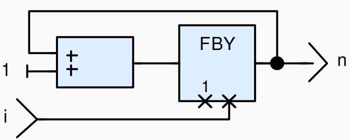

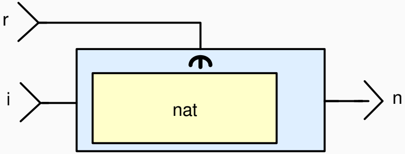

r	F	F	T	F	F
i	0	5	10	15	20
$nat(i)$	0	1	2	3	4
(**restart** nat **every** r)(i)	0	1	10	11	12

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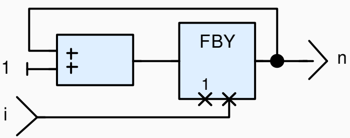

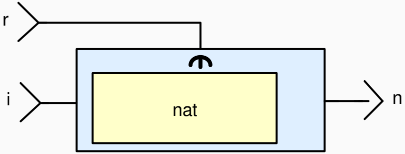

r	F	F	T	F	F	T
i	0	5	10	15	20	25
$nat(i)$	0	1	2	3	4	5
(**restart** nat **every** r)(i)	0	1	10	11	12	25

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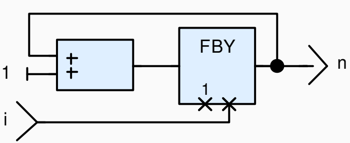

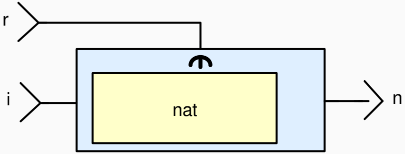

r	F	F	T	F	F	T	F
i	0	5	10	15	20	25	30
$nat(i)$	0	1	2	3	4	5	6
(**restart** *nat* **every** *r*)(*i*)	0	1	10	11	12	25	26

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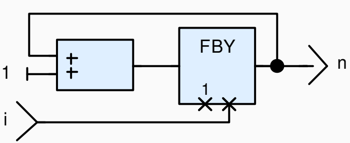

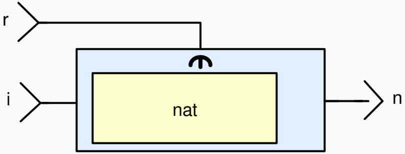

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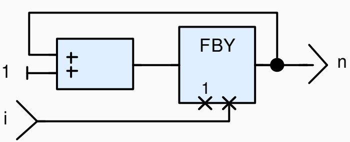
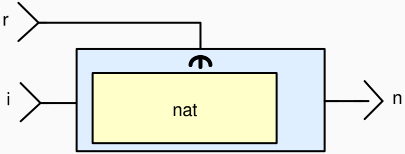
i	0	5	10	15	20	25	30	$\cdots$
$nat(i)$	0	1	2	3	4	5	6	$\cdots$
(**restart** *nat* **every** *r*)(*i*)	0	1	10	11	12	25	26	$\cdots$

A Simpler Example: Intuitive Semantics

[Hamon and Pouzet (2000)]

```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```





r	F	F	T	F	F	T	F	⋯
i	0	5	10	15	20	25	30	⋯
$nat(i)$	0	1	2	3	4	5	6	⋯
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1	10	11	12	25	26	⋯

Could be implemented in a higher-order recursive language

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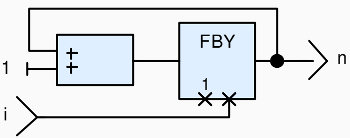

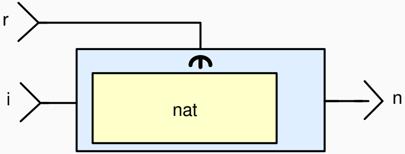

r	F	F	T	F	F	T	F	···
i	0	5	10	15	20	25	30	···
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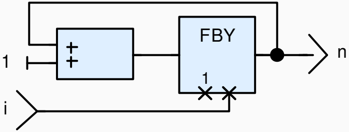

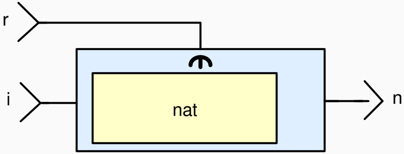

r	F	F	T	F	F	T	F	...
i	0	5	10	15	20	25	30	...
nat(*i*)	0	1	2	3	4	5	6	...
(**restart** *nat* **every** *r*)(*i*)	0	1	10	11	12	25	26	...

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r	F	F	T	F	F	T	F	$\cdots$
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A Simpler Example: Intuitive Semantics

r	F	F	T	F	F	T	F	$\cdots$
count r	0	0	1	1	1	2	2	$\cdots$
i	0	5	10	15	20	25	30	$\cdots$
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1	10	11	12	25	26	$\cdots$

A Simpler Example: Intuitive Semantics

r	F	F	T	F	F	T	F	$\cdots$
count r	0	0	1	1	1	2	2	$\cdots$
i	0	5	10	15	20	25	30	$\cdots$
$\mathrm{mask}_r^0\ i$	0	5						$\cdots$
(`restart` *nat* `every` r)(i)	0	1	10	11	12	25	26	$\cdots$

A Simpler Example: Intuitive Semantics

r	F	F	T	F	F	T	F	$\cdots$
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$\text{mask}_r^0\ i$	0	5						$\cdots$
$nat(\ \text{mask}_r^0\ i)$	0	1						$\cdots$
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1	10	11	12	25	26	$\cdots$

A Simpler Example: Intuitive Semantics

r	F	F	T	F	F	T	F	$\cdots$
count r	0	0	1	1	1	2	2	$\cdots$
i	0	5	10	15	20	25	30	$\cdots$
$\mathrm{mask}_r^0\ i$	0	5						$\cdots$
$nat(\ \mathrm{mask}_r^0\ i)$	0	1						$\cdots$
$\mathrm{mask}_r^1\ i$			10	15	20			$\cdots$
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r	F	F	T	F	F	T	F	$\cdots$
count r	0	0	1	1	1	2	2	$\cdots$
i	0	5	10	15	20	25	30	$\cdots$
$\text{mask}_r^0\ i$	0	5						$\cdots$
$nat(\text{ mask}_r^0\ i)$	0	1						$\cdots$
$\text{mask}_r^1\ i$			10	15	20			$\cdots$
$nat(\text{ mask}_r^1\ i)$			10	11	12			$\cdots$
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r	F	F	T	F	F	T	F	$\cdots$
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$\text{mask}_r^0\ i$	0	5						$\cdots$
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$\text{mask}_r^1\ i$			10	15	20			$\cdots$
$nat(\ \text{mask}_r^1\ i\ )$			10	11	12			$\cdots$
$\text{mask}_r^2\ i$						25	30	$\cdots$
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1	10	11	12	25	26	$\cdots$

A Simpler Example: Intuitive Semantics

r	F	F	T	F	F	T	F	$\cdots$
count r	0	0	1	1	1	2	2	$\cdots$
i	0	5	10	15	20	25	30	$\cdots$
$\text{mask}_r^0\ i$	0	5						$\cdots$
$nat(\ \text{mask}_r^0\ i\ )$	0	1						$\cdots$
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$nat(\ \text{mask}_r^2\ i\ )$						25	26	$\cdots$
$(\texttt{restart}\ nat\ \texttt{every}\ r)(i)$	0	1	10	11	12	25	26	$\cdots$

A Simpler Example: Intuitive Semantics

r	F	F	T	F	F	T	F	$\cdots$
count r	0	0	1	1	1	2	2	$\cdots$
i	0	5	10	15	20	25	30	$\cdots$
$\mathrm{mask}_r^0\ i$	0	5						$\cdots$
$nat(\ \mathrm{mask}_r^0\ i\,)$	0	1						$\cdots$
$\mathrm{mask}_r^1\ i$			10	15	20			$\cdots$
$nat(\ \mathrm{mask}_r^1\ i\,)$			10	11	12			$\cdots$
$\mathrm{mask}_r^2\ i$						25	30	$\cdots$
$nat(\ \mathrm{mask}_r^2\ i\,)$						25	26	$\cdots$
$\vdots$								
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Node instantiation

$$\frac{}{H \vdash_{\text{eqn}} \boldsymbol{x} = f(\boldsymbol{e})}$$

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Formal semantics

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$$\frac{H \vdash_{\text{exp}} \boldsymbol{e} \Downarrow \boldsymbol{es} \quad \vdash_{\text{node}} f(\boldsymbol{es}) \Downarrow \boldsymbol{xs}}{H \vdash_{\text{eqn}} \boldsymbol{x} = f(\,\boldsymbol{e}\,)}$$

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$$\frac{}{H \vdash_{\text{eqn}} x = (\texttt{restart}\ f\ \texttt{every}\ y)(e)}$$

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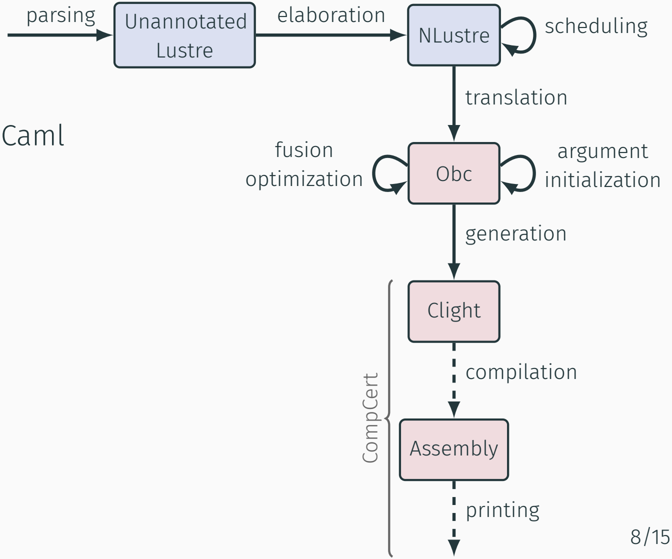
Universally quantified relation: unbounded number of constraints

Vélus: a Verified Lustre Compiler

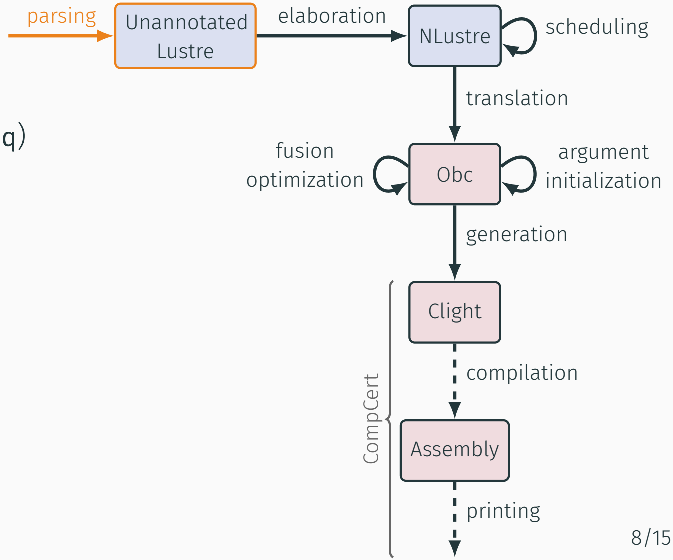
[Bourke et al. (2017)]

Implemented in Coq and (some) OCaml



Vélus: a Verified Lustre Compiler

[Jourdan, Pottier, and Leroy (2012)]



- validated parsing (`menhir --coq`)

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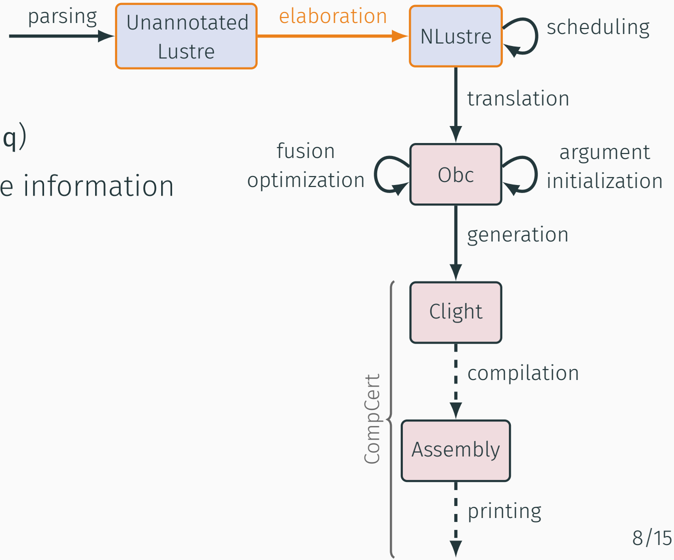

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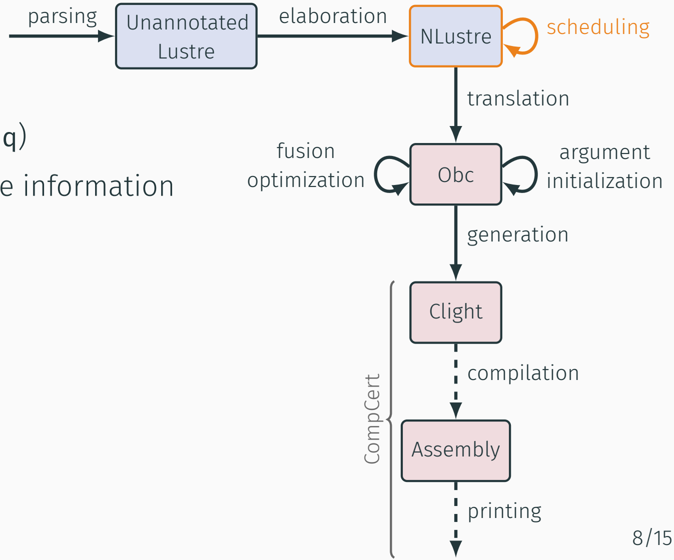
Vélus: a Verified Lustre Compiler



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Vélus: a Verified Lustre Compiler

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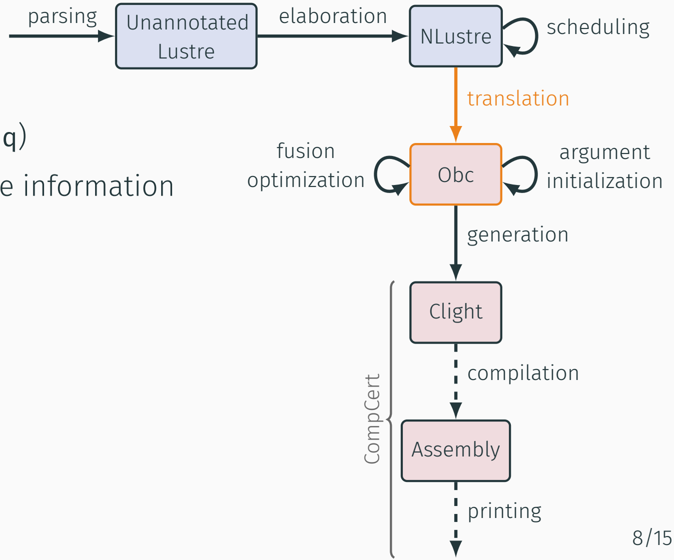

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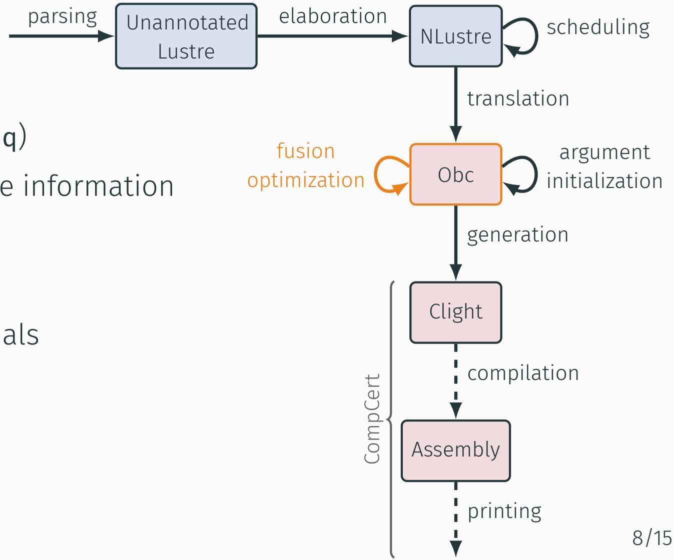

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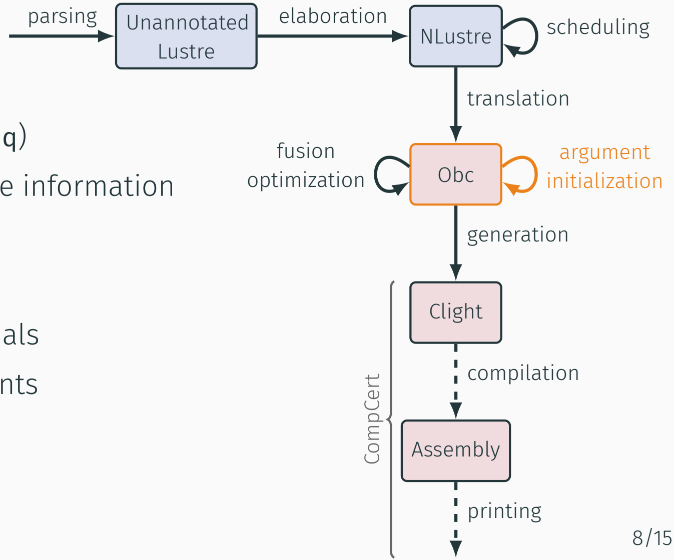

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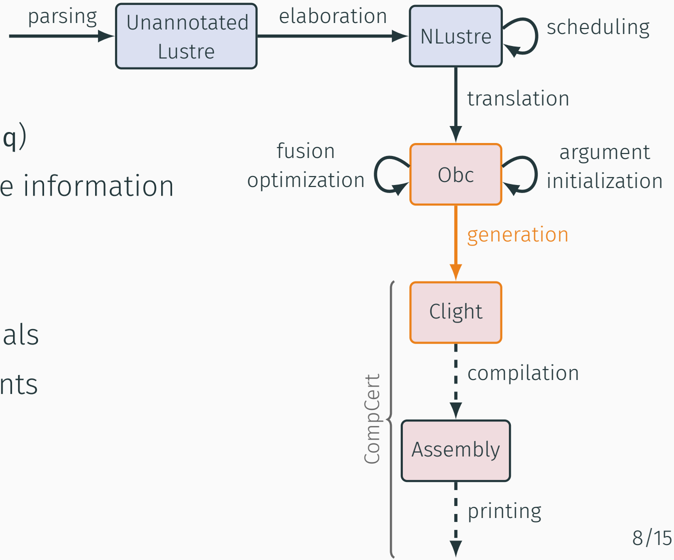
[Blazy and Leroy (2009)]



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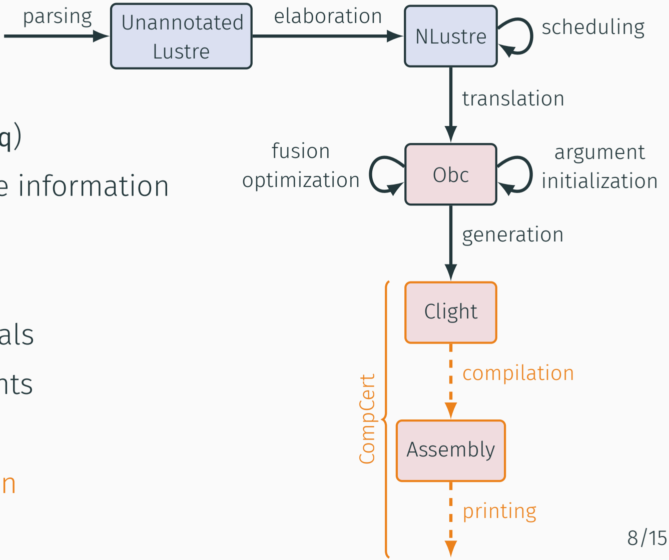
[Leroy (2009)]



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- Rely on CompCert for compilation

A Problem with the Compilation from NLustre to Obc

```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
            ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool
  {
    if r { ins(x).reset() };
    x, ax := ins(x).step(x0, u);
    if r { ins(y).reset() };
    y, ay := ins(y).step(y0, v)
  }
}
```

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Stc: Synchronous Transition Code

Propose a new intermediate language

- Invariant semantics under permutation
- Separate reset construct
- Explicit state: state variables and instances

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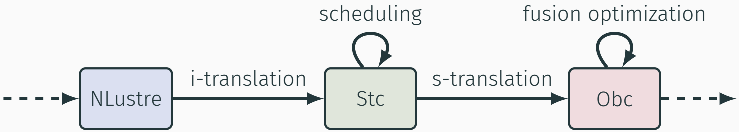
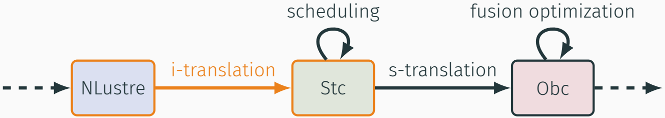


Compilation with Stc



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  var ax, ay: bool;
let
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```

```
system driver {
  sub x: ins, y: ins;

  transition(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool;
  {
    x, ax = ins<x>(x0, u);
    reset ins<x> every (. on r);
    y, ay = ins<y>(y0, v);
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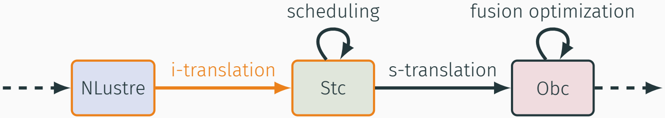
  }
}
```

Compilation with Stc



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  }
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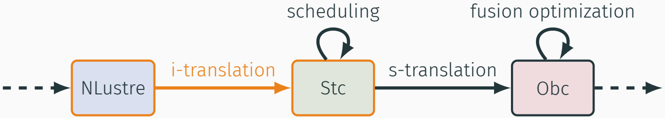
```

Compilation with Stc



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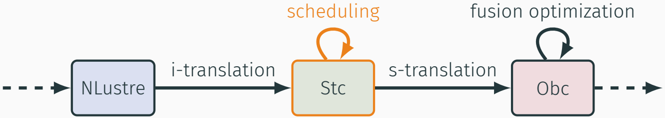
    y, ay = ins<y>(y0, v);
    reset ins<y> every (. on r);
  }
}
```

Compilation with Stc



scheduling

fusion optimization

NLustre

i-translation

Stc

s-translation

Obc

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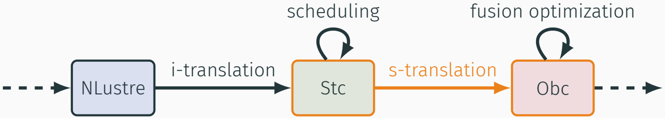
```

```
system driver {
  sub x: ins, y: ins;

  transition(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool;
  {
    reset ins<x> every (. on r);
    reset ins<y> every (. on r);
    x, ax = ins<x>(x0, u);
    y, ay = ins<y>(y0, v);
  }
}
```

Compilation with Stc



```
system driver {
  sub x: ins, y: ins;

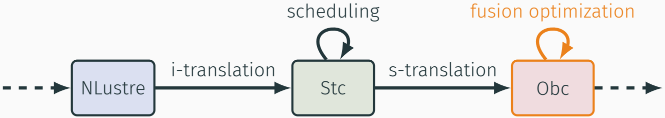
  transition(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool;
  {
    reset ins<x> every (. on r);
    reset ins<y> every (. on r);
    x, ax = ins<x>(x0, u);
    y, ay = ins<y>(y0, v);
  }
}
```

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
            ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool
  {
    if r { ins(x).reset() };
    if r { ins(y).reset() };
    x, ax := ins(x).step(x0, u);
    y, ay := ins(y).step(y0, v)
  }
}
```

Compilation with Stc



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Stc Intuitive Semantics

Transition system

- Start state S, end state S'
- Transition constraints
- Transient state I

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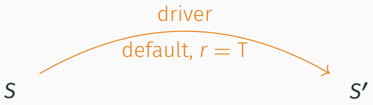

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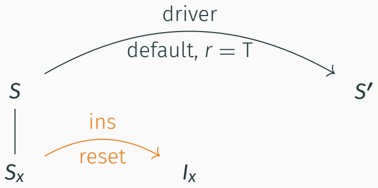

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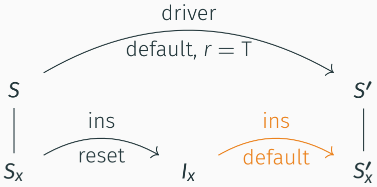

driver

default, $r = \top$

S

S'

ins

reset

ins

default

S_x

I_x

S'_x

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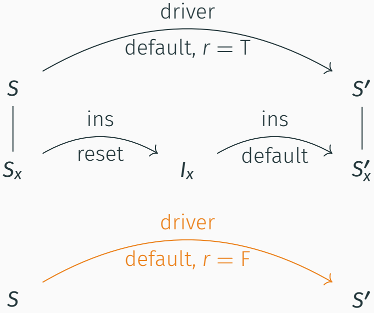

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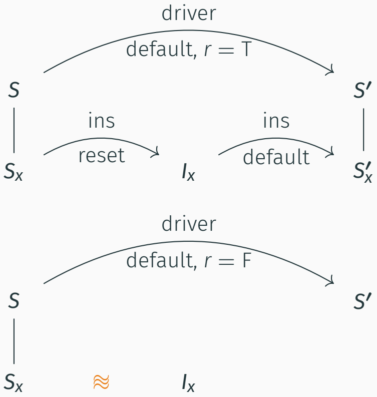

STC INTUITIVE SEMANTICS

Transition system

- Start state S, end state S'
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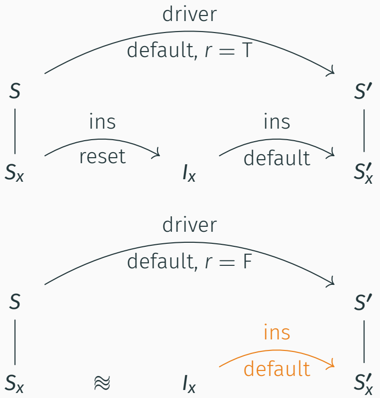
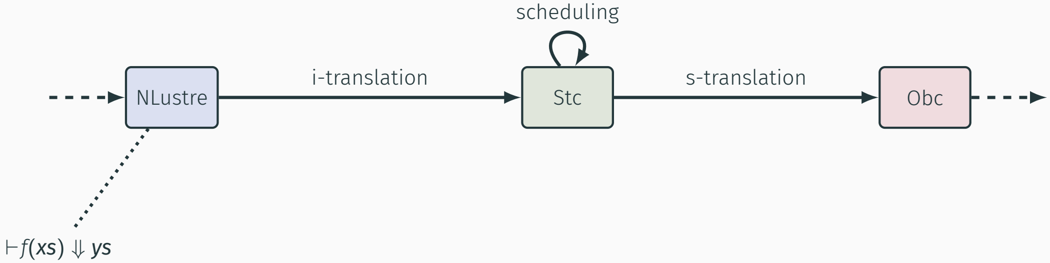


Correctness: Preservation of the Semantics



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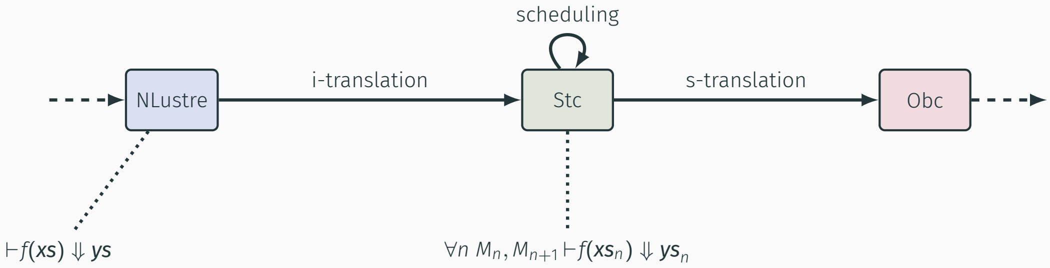
scheduling
NLustre
i-translation
Stc
s-translation
Obc
$\vdash f(xs) \Downarrow ys$
$\forall n\ M_n, M_{n+1} \vdash f(xs_n) \Downarrow ys_n$

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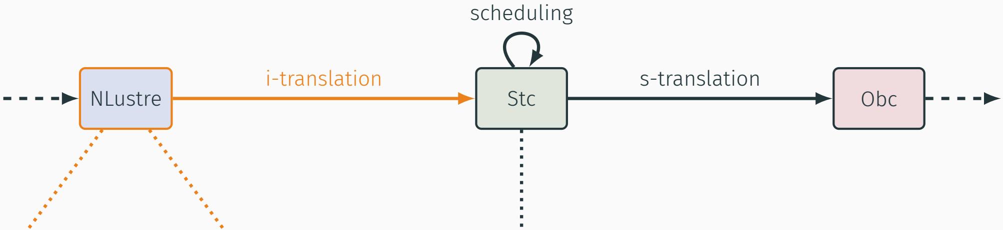

$\vdash f(xs) \Downarrow ys \Longrightarrow \exists M\; M \vdash f(xs) \Downarrow ys$

$\forall n\; M_n, M_{n+1} \vdash f(xs_n) \Downarrow ys_n$

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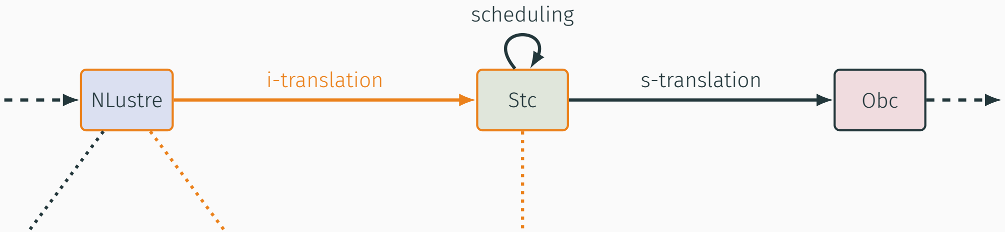

$$\vdash f(xs) \Downarrow ys \Longrightarrow \exists M\ M \vdash f(xs) \Downarrow ys \Longrightarrow \forall n\ M_n, M_{n+1} \vdash f(xs_n) \Downarrow ys_n$$

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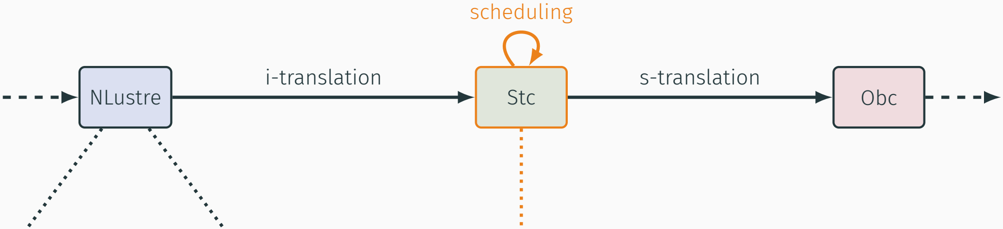

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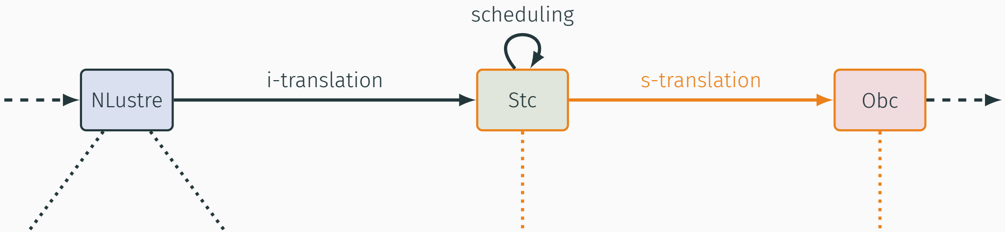

$$\vdash f(xs) \Downarrow ys \Longrightarrow \exists M\ M \vdash f(xs) \Downarrow ys \Longrightarrow \forall n\ M_n, M_{n+1} \vdash f(xs_n) \Downarrow ys_n \Longrightarrow \forall n\ M_n \vdash f.\mathit{step}(xs_n) \Downarrow ys_n, M_{n+1}$$

Ultimate Theorem

Theorem (Vélus correctness)
Given a list of declarations D, a name f, lists of streams of values $\mathbf{xs}$ and $\mathbf{ys}$, an NLustre program G and an assembly program P such that $\mathsf{compile}\,D\,f = \mathsf{OK}\,(G, P)$ and $G \vdash f\,(\mathbf{xs}) \Downarrow \mathbf{ys}$, then there exists an infinite trace of events T such that

$$P \Downarrow_{ASM} \mathsf{Reacts}(T) \quad \textit{and} \quad \mathsf{bisim\text{-}IO}^{G}\, f\, \mathbf{xs}\, \mathbf{ys}\, T$$

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Given a list of declarations D, a name f, lists of streams of values $\mathbf{xs}$ and $\mathbf{ys}$, an NLustre program G and an assembly program P such that $\textsf{compile}\, D\, f = \textsf{OK}\,(G, P)$ and $G \vdash f\,(\mathbf{xs}) \Downarrow \mathbf{ys}$, then there exists an infinite trace of events T such that

$$P \Downarrow_{ASM} \textsf{Reacts}(T) \quad \textit{and} \quad \textsf{bisim-IO}^{G}\, f\; \mathbf{xs}\; \mathbf{ys}\; T$$

$$\begin{tikzcd}
G \arrow[r, "\text{i-tr}"] \arrow[d, Rightarrow, "\text{NLustre}"'] & {} \arrow[r, "\text{sch}"] \arrow[d, Rightarrow, "\text{Stc}"'] & {} \arrow[r, "\text{s-tr}"] \arrow[d, Rightarrow, "\text{Stc}"'] & {} \arrow[r, "\text{fuse}"] \arrow[d, Rightarrow, "\text{Obc}"'] & {} \arrow[r, "\text{init-args}"] \arrow[d, Rightarrow, "\text{Obc}"'] & {} \arrow[r, "\text{gen}"] \arrow[d, Rightarrow, "\text{Obc}"'] & {} \arrow[r, "\text{cl-to-asm}"] \arrow[d, Rightarrow, "\text{Clight}"'] & P \arrow[d, Rightarrow, "\text{Assembly}"'] \\
\mathbf{xs}, \mathbf{ys} \arrow[r, squiggly, dash] \arrow[rrrrrrr, bend right, squiggly, dash, "\text{bisim-IO}"'] & {} \arrow[r, equal] & {} \arrow[r, equal] & {} \arrow[r, equal] & {} \arrow[r, equal] & {} \arrow[r, phantom, "\approx"] & {} \arrow[r, equal] & T
\end{tikzcd}$$

Conclusion

Contributions:

- A verified compiler for Lustre with reset
- A single additional semantic rule for the reset
- An intermediate transition system language: Stc

Next goal: State machines

`velus.inria.fr`
`github.com/INRIA/velus`

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