Verified Lustre Normalization with Node Subsampling

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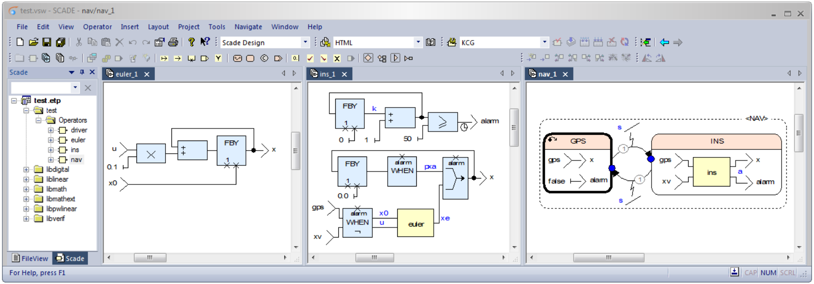
Tuesday, October 12
11:00am - 11:15am EDT

Block-Diagram Languages for Embedded Systems



- Widely used in safety-critical applications: Aerospace, Defense, Rail Transportation, Heavy Equipment, Energy, Nuclear. . .
- Scade 6: Qualified compiler for a Lustre-like language
- Our work: Verified compilation in an Interactive Theorem Prover (Coq)

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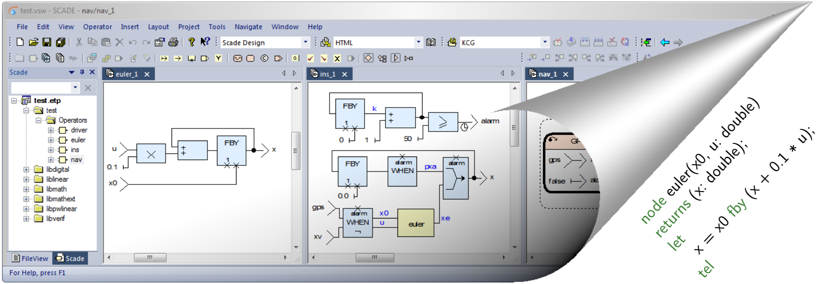

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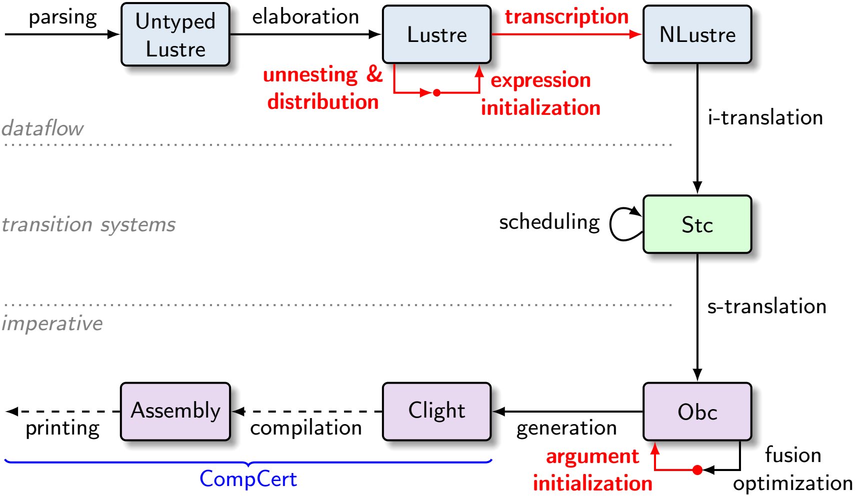

parsing
Untyped Lustre
elaboration
Lustre
transcription
NLustre
unnesting & distribution
expression initialization
dataflow
i-translation
transition systems
scheduling
Stc
s-translation
imperative
Assembly
Clight
Obc
printing
compilation
generation
argument initialization
fusion optimization
CompCert

Lustre: example

"**count down** from *n*, restarting every time *res* is true."

```
node count_down(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n else (n fby (cpt − 1));
tel
```

Lustre: example

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let
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tel
```

res	F	F	F	T	F	F	F	F	F	⋯
n	6	6	6	6	6	6	6	6	6	⋯

Lustre: example

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```

res	F	F	F	T	F	F	F	F	F	⋯
n	6	6	6	6	6	6	6	6	6	⋯
cpt	6	5	4	6	5	4	3	2	1	⋯

Lustre: example

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node count_down(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n else (n fby (cpt − 1));
tel
```

```
node count_down(res : bool; n : int)
returns (cpt : int)
var norm1$1 : int;
let
  norm1$1 = n fby (cpt − 1);
  cpt = if res then n else norm1$1;
tel
```

res	F	F	F	T	F	F	F	F	F	···
n	6	6	6	6	6	6	6	6	6	···
norm1$1	6	5	4	3	5	4	3	2	1	···
cpt	6	5	4	6	5	4	3	2	1	···

Lustre: example

"**count down** from *n*, restarting every time *res* is true."

```
node count_down(res : bool; n : int)
returns (cpt : int)
let
  cpt = if res then n else (n fby (cpt − 1));
tel
```

```
node count_down(res : bool; n : int)
returns (cpt : int)
var norm1$1, norm2$2 : int; norm2$1 : bool;
let
  norm2$1 = true fby false;
  norm2$2 = 0 fby (cpt − 1);
  norm1$1 = if norm2$1 then n else norm2$2;
  cpt = if res then n else norm1$1;
tel
```

res	F	F	F	T	F	F	F	T	F	···
n	6	6	6	6	6	6	6	6	6	···
norm2$1	T	F	F	F	F	F	F	F	F	···
norm2$2	0	5	4	3	5	4	3	2	1	···
norm1$1	6	5	4	3	5	4	3	2	1	···
cpt	6	5	4	6	5	4	3	2	1	···

Unnesting & Distribution function

$$\lfloor \mathsf{c} \rfloor = ([\mathsf{c}], [\,])$$
$$\lfloor \mathsf{x} \rfloor = ([\mathsf{x}], [\,])$$

Unnesting & Distribution function

$$
\begin{aligned}
\lfloor \mathsf{c} \rfloor &= ([\mathsf{c}], [\,]) \\
\lfloor \mathsf{x} \rfloor &= ([\mathsf{x}], [\,]) \\
\lfloor e_1 \oplus e_2 \rfloor &= ([e_1'], \mathsf{eqs}_1') \leftarrow \lfloor e_1 \rfloor \\
&\quad\ ([e_2'], \mathsf{eqs}_2') \leftarrow \lfloor e_2 \rfloor \\
&\quad\ ([e_1' \oplus e_2'], \mathsf{eqs}_1' \ \cup\ \mathsf{eqs}_2')
\end{aligned}
$$

Unnesting & Distribution function

$$
\begin{aligned}
\lfloor c \rfloor &= ([c], [\,]) \\
\lfloor x \rfloor &= ([x], [\,]) \\
\lfloor e_1 \oplus e_2 \rfloor &= ([e_1'], \mathrm{eqs}_1') \leftarrow \lfloor e_1 \rfloor \\
&\quad ([e_2'], \mathrm{eqs}_2') \leftarrow \lfloor e_2 \rfloor \\
&\quad ([e_1' \oplus e_2'], \mathrm{eqs}_1' \cup \mathrm{eqs}_2') \\
\lfloor (e_1, \dots, e_n) \text{ fby } (f_1, \dots, f_m) \rfloor &= ([e_1', \dots, e_k'], \mathrm{eqs}_1') \leftarrow \lfloor e_1, \dots, e_n \rfloor \\
&\quad ([f_1', \dots, f_k'], \mathrm{eqs}_2') \leftarrow \lfloor f_1, \dots, f_m \rfloor \\
&\quad ([x_1, \dots, x_k], [x_1 = e_1' \text{ fby } f_1', \dots, x_k = e_k' \text{ fby } f_k'] \cup \mathrm{eqs}_1' \cup \mathrm{eqs}_2')
\end{aligned}
$$

```
(x, y) = if res
         then (0, 0)
         else ((0, 0) fby (x + 1, y − 1));
```

```
t1 = 0 fby (x + 1);
t2 = 0 fby (y − 1);
x = if res then 0 else t1;
y = if res then 0 else t2;
```

Unnesting & Distribution function

$$
\begin{aligned}
\lfloor c \rfloor &= ([c], [\,]) \\
\lfloor x \rfloor &= ([x], [\,]) \\
\lfloor e_1 \oplus e_2 \rfloor &= ([e_1'], \mathsf{eqs}_1') \leftarrow \lfloor e_1 \rfloor \\
&\quad ([e_2'], \mathsf{eqs}_2') \leftarrow \lfloor e_2 \rfloor \\
&\quad ([e_1' \oplus e_2'], \mathsf{eqs}_1' \cup \mathsf{eqs}_2') \\
\lfloor (e_1, \dots, e_n) \ \mathsf{fby}\ (f_1, \dots, f_m) \rfloor &= ([e_1', \dots, e_k'], \mathsf{eqs}_1') \leftarrow \lfloor e_1, \dots, e_n \rfloor \\
&\quad ([f_1', \dots, f_k'], \mathsf{eqs}_2') \leftarrow \lfloor f_1, \dots, f_m \rfloor \\
&\quad ([x_1, \dots, x_k], [x_1 = e_1' \ \mathsf{fby}\ f_1', \dots, x_k = e_k' \ \mathsf{fby}\ f_k'] \cup \mathsf{eqs}_1' \cup \mathsf{eqs}_2') \\
\lfloor f(e_1, \dots, e_n) \rfloor &= ([e_1', \dots, e_m'], \mathsf{eqs}') \leftarrow \lfloor e_1, \dots, e_n \rfloor \\
&\quad ([x_1, \dots, x_k], [(x_1, \dots x_k) = f(e_1', \dots, e_m')] \cup \mathsf{eqs}')
\end{aligned}
$$

```
(x, y) = if res
         then (0, 0)
         else ((0, 0) fby (x + 1, y − 1));
```

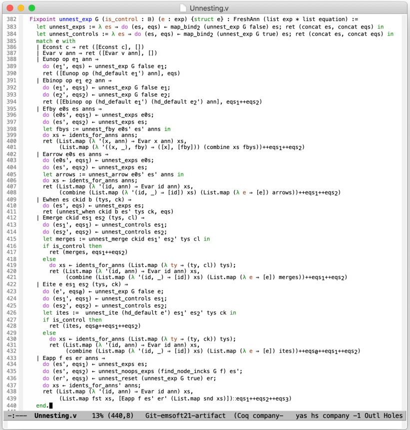
```
t1 = 0 fby (x + 1);
t2 = 0 fby (y − 1);
x = if res then 0 else t1;
y = if res then 0 else t2;
```

Unnesting & Distribution in the Coq Proof Assistant

Unnesting.v

```
Fixpoint unnest_exp G (is_control : 𝔹) (e : exp) {struct e} : FreshAnn (list exp * list equation) :=
  let unnest_exps := λ es ⇒ do (es, eqs) ← map_bind2 (unnest_exp G false) es; ret (concat es, concat eqs) in
  let unnest_controls := λ es ⇒ do (es, eqs) ← map_bind2 (unnest_exp G true) es; ret (concat es, concat eqs) in
  match e with
   | Econst c ⇒ ret ([Econst c], [])
   | Evar v ann ⇒ ret ([Evar v ann], [])
   | Eunop op e1 ann ⇒
     do (e1', eqs) ← unnest_exp G false e1;
     ret ([Eunop op (hd_default e1') ann], eqs)
   | Ebinop op e1 e2 ann ⇒
     do (e1', eqs1) ← unnest_exp G false e1;
     do (e2', eqs2) ← unnest_exp G false e2;
     ret ([Ebinop op (hd_default e1') (hd_default e2') ann], eqs1++eqs2)
   | Efby e0s es anns ⇒
     do (e0s', eqs1) ← unnest_exps e0s;
     do (es', eqs2) ← unnest_exps es;
     let fbys := unnest_fby e0s' es' anns in
     do xs ← idents_for_anns anns;
     ret (List.map (λ '(x, ann) ⇒ Evar x ann) xs,
          (List.map (λ '((x, _), fby) ⇒ ([x], [fby])) (combine xs fbys))++eqs1++eqs2)
   | Earrow e0s es anns ⇒
     do (e0s', eqs1) ← unnest_exps e0s;
     do (es', eqs2) ← unnest_exps es;
     let arrows := unnest_arrow e0s' es' anns in
     do xs ← idents_for_anns anns;
     ret (List.map (λ '(id, ann) ⇒ Evar id ann) xs,
          (combine (List.map (λ '(id, _) ⇒ [id]) xs) (List.map (λ e ⇒ [e]) arrows))++eqs1++eqs2)
   | Ewhen es ckid b (tys, ck) ⇒
     do (es', eqs) ← unnest_exps es;
     ret (unnest_when ckid b es' tys ck, eqs)
   | Emerge ckid es1 es2 (tys, cl) ⇒
     do (es1', eqs1) ← unnest_controls es1;
     do (es2', eqs2) ← unnest_controls es2;
     let merges := unnest_merge ckid es1' es2' tys cl in
     if is_control then
       ret (merges, eqs1++eqs2)
     else
       do xs ← idents_for_anns (List.map (λ ty ⇒ (ty, cl)) tys);
       ret (List.map (λ '(id, ann) ⇒ Evar id ann) xs,
            (combine (List.map (λ '(id, _) ⇒ [id]) xs) (List.map (λ e ⇒ [e]) merges))++eqs1++eqs2)
   | Eite e es1 es2 (tys, ck) ⇒
     do (e', eqs0) ← unnest_exp G false e;
     do (es1', eqs1) ← unnest_controls es1;
     do (es2', eqs2) ← unnest_controls es2;
     let ites :=  unnest_ite (hd_default e') es1' es2' tys ck in
     if is_control then
       ret (ites, eqs0++eqs1++eqs2)
     else
       do xs ← idents_for_anns (List.map (λ ty ⇒ (ty, ck)) tys);
       ret (List.map (λ '(id, ann) ⇒ Evar id ann) xs,
            (combine (List.map (λ '(id, _) ⇒ [id]) xs) (List.map (λ e ⇒ [e]) ites))++eqs0++eqs1++eqs2)
   | Eapp f es er anns ⇒
     do (es', eqs1) ← unnest_exps es;
     do (es', eqs2) ← unnest_noops_exps (find_node_incks G f) es';
     do (er', eqs3) ← unnest_reset (unnest_exp G true) er;
     do xs ← idents_for_anns' anns;
     ret (List.map (λ '(id, ann) ⇒ Evar id ann) xs,
          (List.map fst xs, [Eapp f es' er' (List.map snd xs)])::eqs1++eqs2++eqs3)
  end.
```

-:--- Unnesting.v   13% (440,8)   Git-emsoft21-artifact (Coq company-   yas hs company -1 Outl Holes



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    do (e0s', eqs1) ← unnest_exps e0s;
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    do xs ← idents_for_anns anns;
    ret (List.map (λ '(x, ann) ⇒ Evar x ann) xs,
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    ret (List.map (λ '(id, ann) ⇒ Evar id ann) xs,
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    if is_control then
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    do (es', eqs1) ← unnest_exps es;
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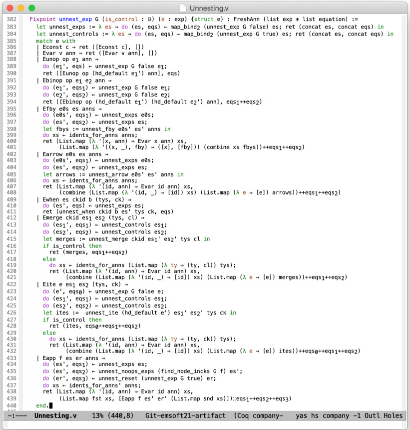
    do (er', eqs3) ← unnest_reset (unnest_exp G true) er;
    do xs ← idents_for_anns' anns;
    ret (List.map (λ '(id, ann) ⇒ Evar id ann) xs,
         (List.map fst xs, [Eapp f es' er' (List.map snd xs)])::eqs1++eqs2++eqs3)
  end.
-:---  Unnesting.v    13% (440,8)   Git-emsoft21-artifact  (Coq company-    yas hs company -1 Outl Holes
```

Fresh identifier generation

- In OCaml:

```
let next = ref 0;;
let fresh () =
    next := !next + 1;
    "norm$"^(string_of_int !next);;
```



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- In OCaml:

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let fresh () =
    next := !next + 1;
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```

- We are in a pure functional language
- Use an explicit state (monad)

$$\boxed{st} \longrightarrow \boxed{st \uplus \{(x, b)\}}$$

```
fresh_ident b st = (x, st')
```

$$\boxed{st}$$

```
do x1 ← fresh_ident b1;
```

$$\boxed{st \uplus \{(x1, b1)\}}$$

```
do x2 ← fresh_ident b2;
```

$$\boxed{st \uplus \{(x1, b1)\} \uplus \{(x2, b2)\}}$$

```
Fixpoint unnest_exp (e : exp) : Fresh (list exp * list equation) ann
```

Stream Semantics of Lustre

`res`	F	F	F	T	F	F	F	F	F	$\cdots$
`n`	6	6	6	6	6	6	6	6	6	$\cdots$
`cpt`	6	5	4	6	5	4	3	2	1	$\cdots$

```
every trigger {
  read inputs;
  calculate;
  write outputs;
}
```

Stream Semantics of Lustre

res	F	F	F	T	F	F	F	F	F	...
n	6	6	6	6	6	6	6	6	6	...
cpt	6	5	4	6	5	4	3	2	1	...

```
every trigger {
  read inputs;
  calculate;
  write outputs;
}
```

$$\text{Svar}\ \frac{H(x) = vs}{H \vdash x \Downarrow vs}$$

```
Inductive sem_exp:
  History → exp → list Stream → Prop :=
| Svar: sem_var H x vs →
        sem_exp H (Evar x ann) [vs]        [...]
```

Stream Semantics of Lustre

res	F	F	F	T	F	F	F	F	F	...
n	6	6	6	6	6	6	6	6	6	...
cpt	6	5	4	6	5	4	3	2	1	...

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every trigger {
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```
Inductive sem_exp:
  History → exp → list Stream → Prop :=
| Svar: sem_var H x vs →
        sem_exp H (Evar x ann) [vs]       [...]
```

$$\text{Seq}\ \frac{H \vdash \mathsf{es} \Downarrow H(\mathsf{xs})}{H \vdash \mathsf{xs} = \mathsf{es}}$$

```
with sem_equation:
  History → equation → Prop :=
| Seq: Forall2 (sem_exp H) es ss →
       Forall2 (sem_var H) xs (concat ss) →
       sem_equation H (xs, es)
```

Stream Semantics of Lustre

res	F	F	F	T	F	F	F	F	F	...
n	6	6	6	6	6	6	6	6	6	...
cpt	6	5	4	6	5	4	3	2	1	...

```
every trigger {
  read inputs;
  calculate;
  write outputs;
}
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$$\text{Svar}\ \frac{H(x) = vs}{H \vdash x \Downarrow vs}$$

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Inductive sem_exp:
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  History → equation → Prop :=
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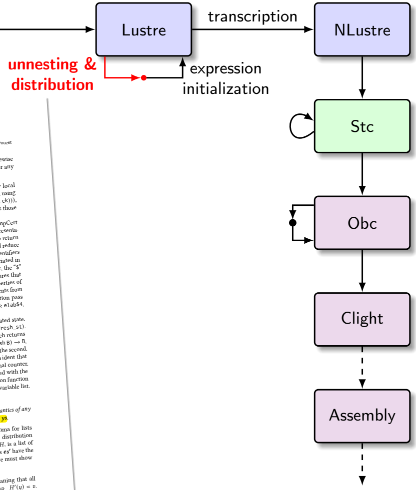
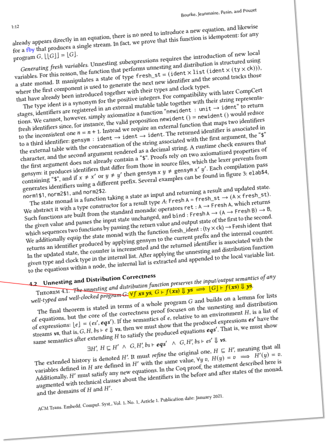
       Forall2 (sem_var H) xs (concat ss) →
       sem_equation H (xs, es)
```

$$\text{Snode}\ \frac{\begin{array}{c}\mathtt{node}(G, f) \doteq n \\ H(n.\mathbf{in}) = \mathrm{xs} \qquad H(n.\mathbf{out}) = \mathrm{ys} \qquad \forall eq \in n.\mathbf{eqs},\ G, H \vdash eq\end{array}}{G \vdash f(\mathrm{xs}) \Downarrow \mathrm{ys}}$$

Unnesting & Distribution – correctness





already appears directly in an equation, there is no need to introduce a new equation, and likewise for a fby that produces a single stream. In fact, we prove that this function is idempotent: for any program G, $\lfloor\lfloor G\rfloor\rfloor = \lfloor G\rfloor$.

Generating fresh variables. Unnesting subexpressions requires the introduction of new local variables. For this reason, the function that performs unnesting and distribution is structured using a state monad. It manipulates a state of type `fresh_st` = (`ident` × `list` (`ident` × (`ty` × `ck`))), where the first component is used to generate the next new identifier and the second tracks those that have already been introduced together with their types and clock types.

The type ident is a synonym for the positive integers. For compatibility with later CompCert stages, identifiers are registered in an external mutable table together with their string representations. We cannot, however, simply axiomatize a function "`newident : unit → ident`" to return fresh identifiers since, for instance, the valid proposition `newident () = newident ()` would reduce to the inconsistent one $n = n + 1$. Instead we require an external function that maps two identifiers to a third identifier: `gensym : ident → ident → ident`. The returned identifier is associated in the external table with the concatenation of the string associated with the first argument, the "`$`" character, and the second argument rendered as a decimal string. A runtime check ensures that the first argument does not already contain a "`$`". Proofs rely on two axiomatized properties of gensym: it produces identifiers that differ from those in source files, which the lexer prevents from containing "`$`", and if $x \neq x'$ or $y \neq y'$ then `gensym` $x\ y \neq$ `gensym` $x'\ y'$. Each compilation pass generates identifiers using a different prefix. Several examples can be found in figure 3: `elab$4`, `norm1$1`, `norm2$1`, and `norm2$2`.

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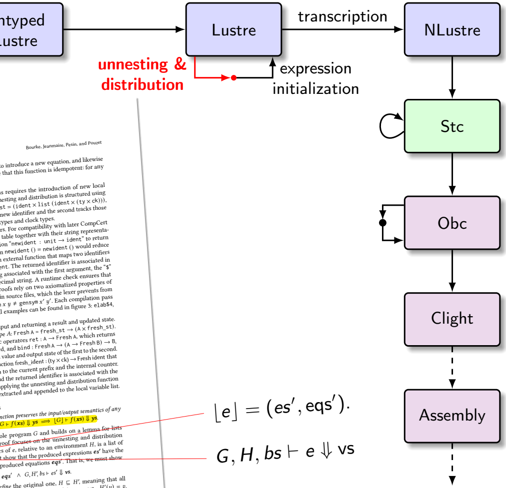
Unnesting & Distribution – correctness



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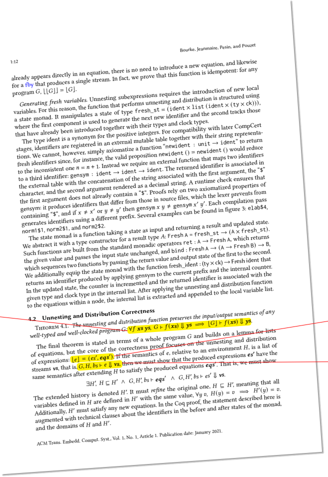

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Bourke, Jeanmaire, Pesin, and Pouzet

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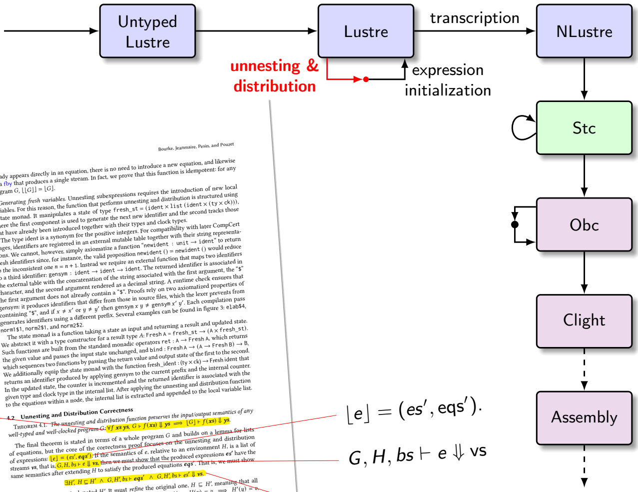

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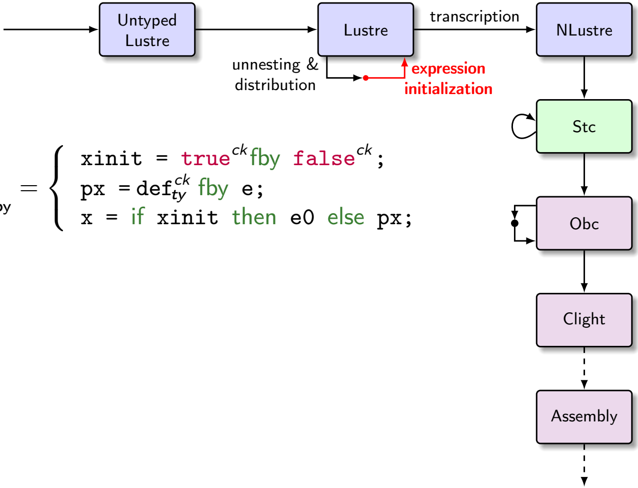

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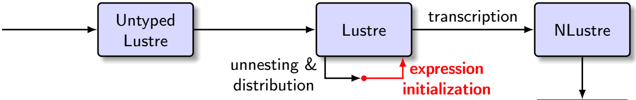

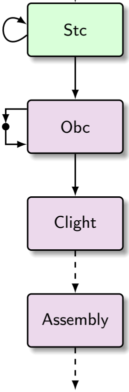

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Optimization: avoid introducing several init registers

- Registers are costly in the final imperative program
- Use state monad to remember init registers introduced:
  `Fresh A (ann * bool)`
- Complicates the correctness proof with a non-local invariant

Clock system correctness

`x = 0 fby (x + 1)`	0	1	2	3	4	5	6	8	9	...
`b`	T	T	F	F	T	T	T	F	F	...
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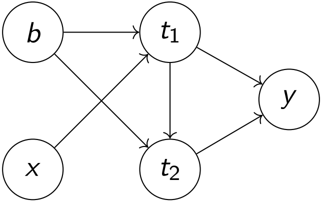
- induction on equations is not enough: `(x, y) = (42, x)`
- causal node → acyclic graph

```
node f(b : bool; x : int) returns (y : int)
var t1, t2 : int;
let
    (t1, t2) = if b
               then (x + 1, t1)
               else (x − 1, −t1);
    y = (0 fby y) + (t1 ∗ t2);
tel
```



- graphs difficult to handle in a proof assistant: 1200 lines of Coq

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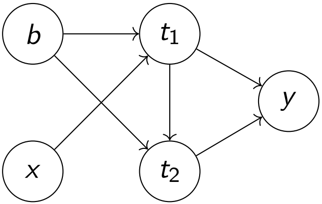

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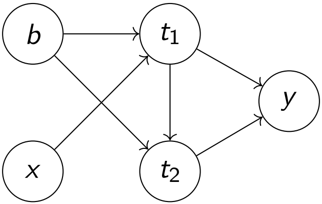

- graphs difficult to handle in a proof assistant: 1200 lines of Coq
- induction on a topological ordering of the nodes of the graph
- look only to the left of fby: the fby operator forces alignment
- intricate proof: around 2000 lines of Coq proof script

Node Subsampling

node current(d : `int`; ck : `bool`; x : `int` when ck)

Node Subsampling

node current($\underbrace{\text{d : int; ck : bool}}_{\text{always present}}$; x : int when ck)

Node Subsampling

$\text{node current}(\underbrace{\text{d} : \texttt{int}; \text{ck} : \texttt{bool}}_{\text{always present}}; \underbrace{\text{x} : \texttt{int} \text{ when ck}}_{\text{only present when ck is}})$

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$$\text{node current}(\underbrace{d : \texttt{int};\ ck : \texttt{bool}}_{\text{always present}};\ \underbrace{x : \texttt{int}\ \text{when}\ ck}_{\text{only present when ck is}})$$

Compile an instance of this node to Obc:

```
if (ck) {
  elab$4 := exp;
};
time := current(i1).step(0, ck, elab$4)
```

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only defined when ck = true

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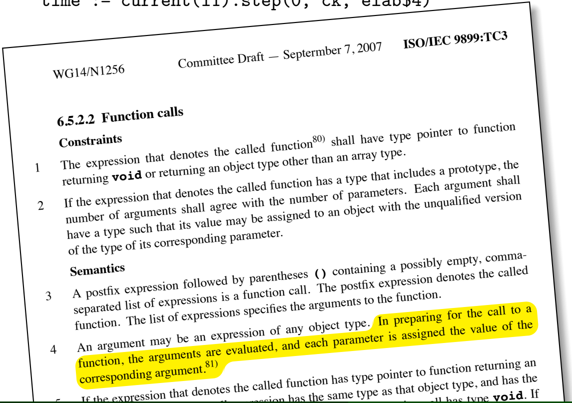

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6.5.2.2 Function calls

Constraints

1 The expression that denotes the called function[80] shall have type pointer to function returning **void** or returning an object type other than an array type.

2 If the expression that denotes the called function has a type that includes a prototype, the number of arguments shall agree with the number of parameters. Each argument shall have a type such that its value may be assigned to an object with the unqualified version of the type of its corresponding parameter.

Semantics

3 A postfix expression followed by parentheses **()** containing a possibly empty, comma-separated list of expressions is a function call. The postfix expression denotes the called function. The list of expressions specifies the arguments to the function.

4 An argument may be an expression of any object type. In preparing for the call to a function, the arguments are evaluated, and each parameter is assigned the value of the corresponding argument.[81]

5 If the expression that denotes the called function has type pointer to function returning an

Node Subsampling

node current($\underbrace{\text{d : int; ck : bool;}}_{\text{always present}}$ $\underbrace{\text{x : int when ck}}_{\text{only present when ck is}}$)

Compile an instance of this node to Obc:

```
if (ck) {
  elab$4 := exp;
};
time := current(i1).step(0, ck, elab$4)
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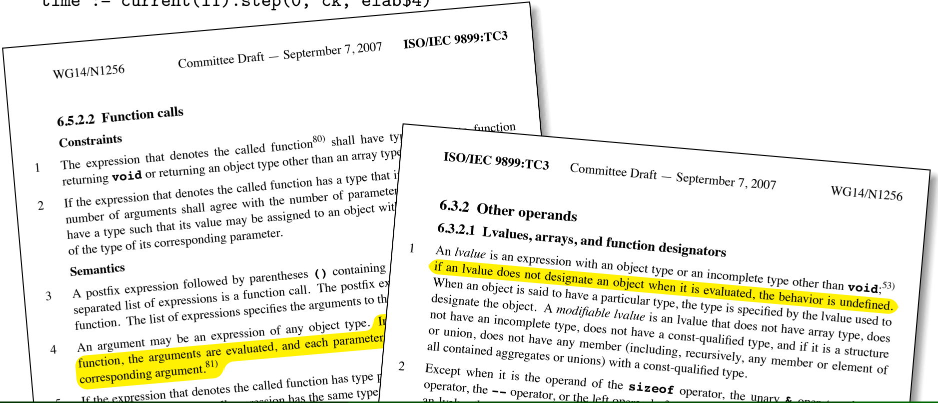
only defined when ck = true



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ISO/IEC 9899:TC3 Committee Draft — September 7, 2007 WG14/N1256

6.3.2 Other operands

6.3.2.1 Lvalues, arrays, and function designators

1 An *lvalue* is an expression with an object type or an incomplete type other than **void**;[53] if an lvalue does not designate an object when it is evaluated, the behavior is undefined. When an object is said to have a particular type, the type is specified by the lvalue used to designate the object. A *modifiable lvalue* is an lvalue that does not have array type, does not have an incomplete type, does not have a const-qualified type, and if it is a structure or union, does not have any member (including, recursively, any member or element of all contained aggregates or unions) with a const-qualified type.

2 Except when it is the operand of the **sizeof** operator, the unary **&** oper... operator, the **--** operator, or the left opera...

Node Subsampling

node current($\underbrace{\texttt{d : int; ck : bool}}_{\text{always present}}$; $\underbrace{\texttt{x : int when ck}}_{\text{only present when ck is}}$)

Compile an instance of this node to Obc:

```
if (ck) {
  elab$4 := exp;
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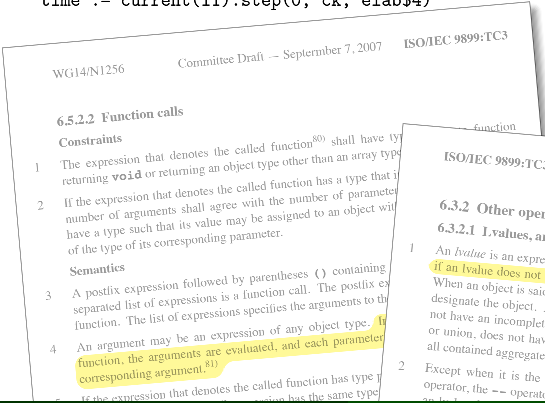
only defined when ck = true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4 An argument may be an expression of any object type. In function, the arguments are evaluated, and each parameter corresponding argument.[81]

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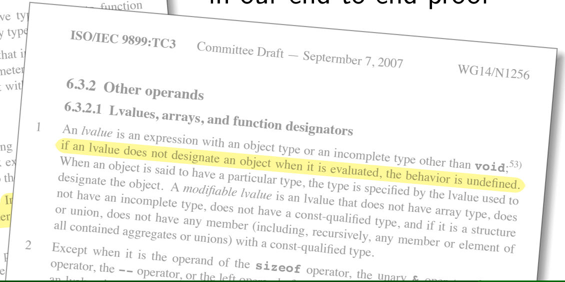
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2 Except when it is the operand of the **sizeof** operator, the unary **&** operator, the **--** operator, or the left operand

- Already formalized in CompCert's Clight semantics
- Appears as a proof obligation in our end-to-end proof

Node Subsampling

node current$(\underbrace{\text{d : int; ck : bool}}_{\text{always present}};\ \underbrace{\text{x : int when ck}}_{\text{only present when ck is}})$

Compile an instance of this node to Obc:

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if (ck) {
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```

1. Add validity assertions during compilation:

```
if (ck) {
  elab$4 := exp;
};
time := current(i1).step(0, ⟨ck⟩, elab$4)
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time := current(i1).step(0, ⟨ck⟩, elab$4)
```

2. Extra compilation pass to initialize variables:

```
if (ck) {
  elab$4 := exp;
} else {
  elab$4 := 0;
};
time := current(i1).step(0, ⟨ck⟩, ⟨elab$4⟩)
```

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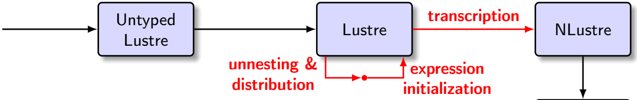
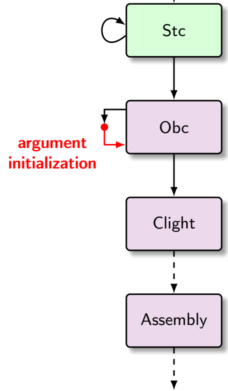
```

- Guarantees that variables in function calls are always defined.
- Recover Obc $\rightsquigarrow$ Clight proof
- Programs without subsampling are unchanged

Conclusion



res	F	F	F	T	F	F	F	F	F	···
n	6	6	6	6	6	6	6	6	6	···
cpt	6	5	4	6	5	4	3	2	1	···



- More expressive source language and associated semantic model
- Formally verified normalization algorithm
  - » Separate concerns and proofs over 3 functions
  - » Requires correctness of clock system
- Allow node subsampling in source language
  - » Add “validity assertions” and explicit initialization
- **End-to-end machine-checked proof connecting the dataflow semantics of an expressive source language with the low-level assembly semantics.**
- Source code and online demo: https://velus.inria.fr

```
every trigger {
  read inputs;
  calculate;
  write outputs;
}
```